Math 5 – Trigonometry – Chapter 5 Test – fall '08 Name__________________________

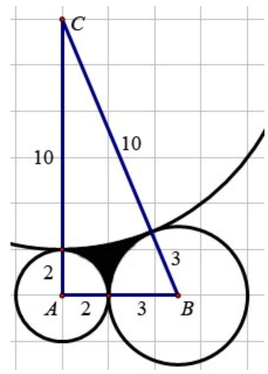
Show your work for credit. Write all responses on separate paper.

1. What is the central angle subtended by an arc length of 3π in a circle of radius 15? Write the radian measure of the angle.

2. Find the arc length in a circle of radius 4 that is subtended by an angle with radian measure 3.

3. What is the radius of a circle where a sector with central angle of 72˚ has area = π?

4. Three circles with radii 2cm, 3cm and 10cm are externally tangent to one another, as shown in the figure at right.



   a. Show that triangle ABC is a right triangle.

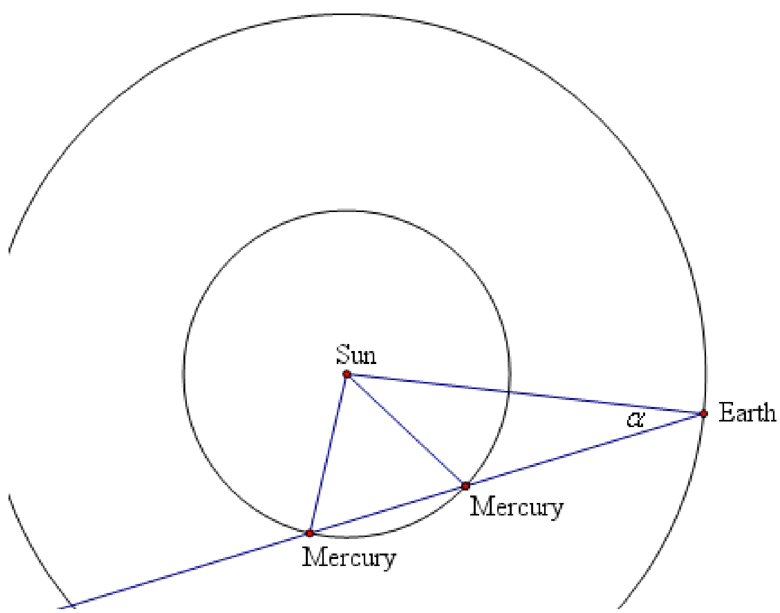
   b. Approximate to the nearest hundredth of a degree measures for $\angle B$ and $\angle C$, interior to triangle ABC.

   c. Approximate the area of the shaded region between the three circles to the nearest hundredth of a square cm.

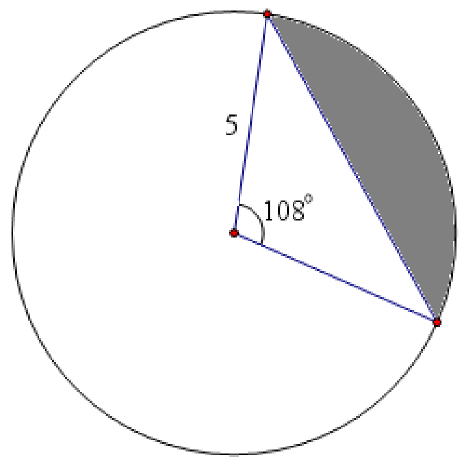
5. A bicycle with wheels of radius 0.3 meters is rolling at a linear speed of 114 meters per minute.
   a. What is the angular speed of the wheel is rad/min?
   b. How many times do the wheels rotate in 10 minutes?

6. The elongation α for Mercury is the angle formed by the planet, Earth and Sun, as shown in the diagram at right. Assume the distance from Mercury to the sun is 0.387 AU (38.7% of the distance from Earth to Sun) and that α = 18˚. Since this is an ASS situation, there are two triangles which satisfy these conditions, as shown at right.

   Find both possible distances from Earth to Mercury.



7. Find the area of the shaded region in the figure at right.



8. Approximate to the nearest hundredth of a degree, the interior angles of a triangle with sides 4, 5 and 6.

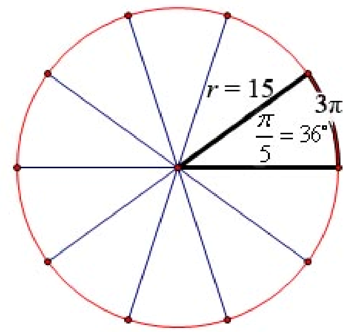
9. Suppose an interior angle of a triangle measures 1 rad. and is nested between sides of lengths 11 and 14. What is the length of the side opposite that angle?

10. Suppose we have vectors $\vec{u} = \langle 1,4 \rangle$ and $\vec{v} = 2\hat{i} - 3\hat{j}$
    a. Draw and label these vectors together in the *x*-*y* plane, assuming each has its initial point at (0,0).
    b. Find the length of $\vec{u}$ and the length of $\vec{v}$.
    c. Find the lengths of $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$
    d. Find the angle between these two vectors.
    Use the formula $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

Math 5 – Trigonometry – Chapter 5 Test Solutions – Fall ‘08.

1. What is the central angle subtended by an arc length of 3π in a circle of radius 15? Write the radian measure of the angle.

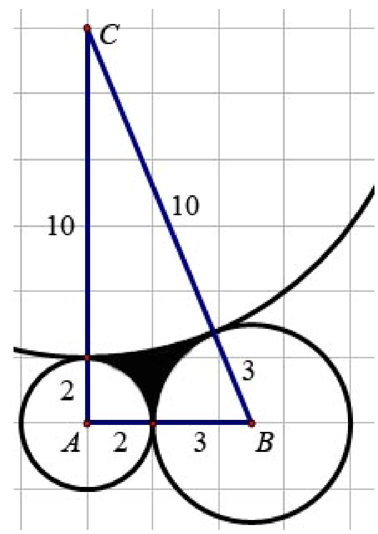
   SOLN: Using the formula $s = r\theta$, we have $3\pi = 15\theta \Rightarrow \boxed{\theta = \frac{\pi}{5}}$ $\frac{\pi}{5} = 36^\circ$



2. Find the arc length in a circle of radius 4 that is subtended by an angle with radian measure 3.
   SOLN: Same formula as in #1, but with different givens:
   $s = r\theta = 4*3 = 12$.

3. What is the radius of a circle where a sector with central angle of 72° has area = π?
   SOLN: Use the formula $A = r^2\theta/2$ with $\theta = 72\pi/180 = 2\pi/5$. That is $\pi = r^2\pi/5$ so $\boxed{r = \sqrt{5}}$.

4. Three circles with radii 2cm, 3cm and 10cm are externally tangent to one another, as shown in the figure at right.
   a. Show that triangle ABC is a right triangle.
      SOLN: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$.
   b. Approximate to the nearest hundredth of a degree measures for $\angle B$ and $\angle C$, interior to triangle *ABC*.
      SOLN: $\angle B = \arcsin(12/13) \approx 67.38^\circ$,
      $\angle C \approx 90^\circ - 67.38^\circ = 22.62^\circ$
   c. Approximate the area of the shaded region between the three circles to the nearest hundredth of a square cm.



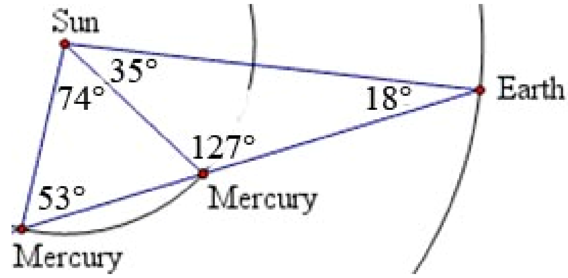
   SOLN: The portion in circle A is simply ¼ its area: π. The area in circle B is approximately 67.38/360 of $9\pi \approx 1.6845\pi$ and the part in circle C is approximately 22.62/360 of $100\pi \approx 6.283\pi$. So the total area is approximately $30 - 8.968\pi \sim 1.83$ cm^2.

5. A bicycle with wheels of radius 0.3 meters is rolling at a linear speed of 114 meters per minute.

   a. What is the angular speed of the wheel is rad/min?
      SOLN: $\frac{114\text{meters}}{\text{min}} \times \frac{2\pi\text{ radians}}{2\pi(0.3)\text{meters}} = 380\frac{\text{rad}}{\text{min}}$

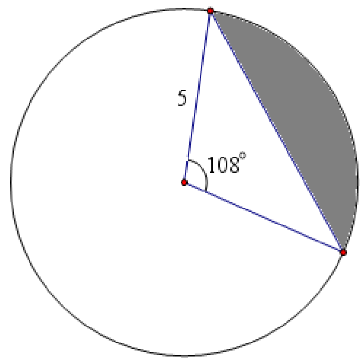
   b. How many times do the wheels rotate in 10 minutes?
      SOLN: $380\frac{\text{rad}}{\text{min}} \times \frac{1\text{revolution}}{2\pi\text{ radians}} \times 10\text{min} = \frac{1900}{\pi} \approx 604.8$ revolutions in 10 minutes.

6. The elongation α for Mercury is the angle formed by the planet, Earth and Sun, as shown in the diagram at right. Assume the distance from Mercury to the sun is 0.387 AU (38.7% of the distance from Earth to Sun) and that α = 18˚. Since this is an ASS situation, there are two triangles which satisfy these conditions, as shown at right.
Find both possible distances from Earth to Mercury.



SOLN: Law of sines tells us that $\frac{\sin 18^\circ}{0.387} = \frac{\sin\angle M}{1} \Rightarrow \angle M = \sin^{-1}\left(\frac{\sin 18^\circ}{0.387}\right) \approx 53^\circ$ so the longer distance is $d \approx \frac{0.387}{\sin 18^\circ}\sin\left(180^\circ - 53^\circ - 18^\circ\right) = \frac{0.387}{\sin 18^\circ}\sin\left(109^\circ\right) \approx 1.184$ AU and the shorter distance is $d \approx \frac{0.387}{\sin 18^\circ}\sin\left(35^\circ\right) \approx 0.718$ AU.

7. Find the area of the shaded region in the figure at right.
SOLN: Area = sector area – triangle area =
$\frac{108}{360}\pi 5^2 - \frac{1}{2}*5*5\sin 108^\circ = \frac{75\pi}{10} - \frac{25}{2}\sin 72^\circ \approx 11.67$

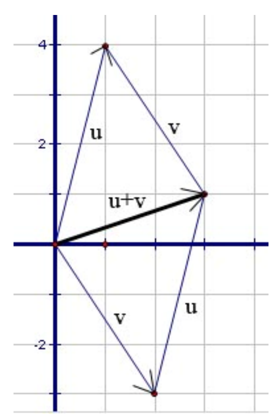


8. Approximate to the nearest hundredth of a degree, the interior angles of a triangle with sides 4, 5 and 6.
SOLN: By the law of cosines, the angle opposite the longest side is $\theta = \cos^{-1}\left(\frac{6^2 - 5^2 - 4^2}{-2*4*5}\right) \approx 82.82^\circ$

By law of sines, the angle opp. 5 = $\sin^{-1}\left(\frac{5\sin 82.82^\circ}{6}\right) \approx 55.77^\circ$ leaving 41.41° for the small angle.

9. Suppose an interior angle of a triangle measures 1 rad. and is nested between sides of lengths 11 and 14. What is the length of the side opposite that angle?
SOLN: By the law of cosines this length is
$\sqrt{11^2 + 14^2 - 2(11)(14)\cos 1} = \sqrt{121 + 196 - 308\cos\frac{180^\circ}{\pi}} \approx 12.27$

10. Suppose we have vectors $\vec{u} = \langle 1,4 \rangle$ and $\vec{v} = 2\hat{i} - 3\hat{j}$



a. Draw and label these vectors together in the *x-y* plane, assuming each has its initial point at (0,0). SOLN: see diagram at right.
b. Find the length of $\vec{u}$ and the length of $\vec{v}$.
SOLN: $|\vec{u}| = \sqrt{1^2 + 4^2} = \sqrt{17}$ and $|\vec{v}| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$
c. Find the lengths of $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$
SOLN: $|\vec{u} + \vec{v}| = |\langle 3,1 \rangle| = \sqrt{3^2 + 1^2} = \sqrt{10}$ and
$|\vec{u} - \vec{v}| = |\langle -1,7 \rangle| = \sqrt{(-1)^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$
d. Find the angle between these two vectors. Use the formula
$\theta = \cos^{-1}\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \cos^{-1}\left(\frac{2-12}{\sqrt{17\cdot 13}}\right) \approx 132.27^\circ$